TWO-PHASE FLOW OF A GAS AND ENTRAINED SOLID PARTICLES PAST A BODY WITH EROSION

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The flow of a mixture of a gas with solid particles is often accompanied by destruction (erosion) of the material of obstructing bodies as a result of numerous particle impacts on the surface. In the course of erosion the shape of the body may change appreciably, causing the entire gasdynamic flow pattern to be restructured. It is therefore necessary to incorporate a description of the erosion process into the conventional scheme for the analysis of two-phase flows. On the other hand, surface wear diminishes the strength characteristics of the structure, posing a serious engineering problem. Several papers [1-4] have been concerned with determining the magnitude of the wear associated with erosion in various aerodynamic situations, but they lack a correct mathematical model of the effect.

In the present article we formulate the boundary conditions for the equations of motion of a dusty gas with direct incorporation of the surface erosion created by particle impingement. Within the framework of the proposed model we investigate the erosion problem for a slender planar profile and the initial stage of erosion of a wedge in a hypersonic dusty gas flow.

1. Boundary Conditions Corresponding to Erosion

The erosion process evolves as a result of the impacts of a large number of small particles against the surface of a solid boundary. In each impact event a certain volume of material is removed from the surface and carried away by the flow. The impact effect of one microscopic particle is microscopic itself, but if the number of impacts is large, the volume of material removed by erosion becomes quite large. Let us suppose that prior to the start of erosion the surface of the solid is described by the equation

$$z - h_0(x,\ y) = 0.$$

During erosion the original surface changes appreciably, so that at each instant the equation for the surface takes the form

$$z - h(x,\ y,\ t) = 0.$$

The mass of material removed by erosion during a time Δt from unit area is

$$\Delta M = \rho_* [h(x, y, t) - h(x, y, t + \Delta t)], \tag{1.1}$$

where ρ_* is the density of the eroded material. If we neglect multiple-impact effects, which is valid for a small volume content of particles, the erosion is the sum of the contributions from the impacts of the individual particles, and the following equation holds:

$$\Delta M = \delta M \dot{n} \Delta t, \tag{1.2}$$

where δM is the mass of material removed in the impact of one particle and $\dot{n}$ is the particle flux onto unit area. The quantity δM can be determined if the mass m_p of one particle and the erosion coefficient E are known, according to the equation

$$\delta M = E m_p. \tag{1.3}$$

Substituting (1.3) into (1.2) and (1.1), expanding the function $h(x, y, t + \Delta t)$ in a Taylor series about the point (x, y, t), and passing to the limit $\Delta t \to 0$, we obtain

$$\rho_* \partial h / \partial t = E \rho_p (\mathbf{v}_p \cdot \mathbf{n}) \sqrt{1 + (\partial h/\partial x)^2 + (\partial h/\partial y)^2}, \tag{1.4}$$

where ρ_p, $\mathbf{v}_p$ are the density and velocity of the particle stream and $\mathbf{n}$ is the unit outward normal to the surface $z = h(x, y, t)$.

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Equation (1.4) relates the parameters of the impingent flow to the variation of the surface itself and the erosion parameters. We have thus reduced the problems of two-phase flow past bodies with erosion to problems involving a variable boundary. In solving the corresponding problem we can extract, together with various values of the field in the flow volume, information about the local wear values $\delta h = h_0(x, y) - h(x, y, t)$, i.e., predict erosion. For this purpose it is necessary to know the erosion coefficient E, which is a function of the impact parameters.

In erosion tests a sample of the investigated material is placed in a dusty gas flow with known parameters. As a result of erosion, the mass of the sample changes by the amount

$$\Delta M = -\Delta t \int_S E\rho_p (\mathbf{v}_p \cdot \mathbf{n})\, dS,$$

and the mass of the abrasive consumed in the erosion process is

$$\Delta m_p = -\Delta t \int_S \rho_p (\mathbf{v}_p \cdot \mathbf{n})\, dS,$$

where S is the area of the eroded surface. From the definition we obtain the relationship of the experimentally determined erosion coefficient $\langle E \rangle = \Delta M/\Delta m_p$ to the coefficient E:

$$\langle E\rangle = \int_S E\rho_p (\mathbf{v}_p \cdot \mathbf{n})\, dS \Big/ \int_S \rho_p (\mathbf{v}_p \cdot \mathbf{n})\, dS. \tag{1.5}$$

When the parameters of the particle stream vary only slightly along the sample, this relation implies $\langle E \rangle = E$. We have thus determined the erosion coefficient used to predict wear in [1, 2]. On the other hand, writing expression (1.4) in increments and solving it for E, we obtain

$$E = -[\rho_* \delta h/\rho_p (\mathbf{v}_p \cdot \mathbf{n})\, \delta t]\,[1 + (\partial h/\partial x)^2 + (\partial h/\partial y)^2]^{-1/2}, \tag{1.6}$$

where $\delta h = h_0(x, y) - h(x, y, \delta t)$ and δt is the erosion time. Consequently, knowing the distribution of the parameters of the particle stream along the sample and the experimental values of the wear δh, we can use Eq. (1.6) to determine the erosion coefficient E.

In the range of medium impact velocities the behavior of the erosion coefficient has been studied in detail for many materials (see, e.g., [6, 7]). For high impact velocities ($v_p \gtrsim 1$ km/sec) we can use single-impact data processed in application to the erosion conditions. In the ensuing calculations we use the following expression for the erosion coefficient:

$$E = \frac{v_p^2}{2H_e} F(\alpha), \quad F(\alpha) = \begin{cases} \frac{4}{3}\left(\sin\alpha - \frac{1}{4}\right), & \alpha > 30°, \\ \frac{4}{3}(\sin\alpha)^2, & \alpha \leqslant 30°. \end{cases} \tag{1.7}$$

Here α is the angle of attack, and H_e is the effective erosion enthalpy [8].

The function $E(\alpha)$ is plotted in Fig. 1 (solid curve) along with experimental data [9] obtained in the impact of steel pellets of millimeter diameter against a lead target. The sets of data correspond to the following impact velocities: 1) 3.19 km/sec; 2) 3.81; 3) 5.01 km/sec. This behavior of the erosion as a function of the angle of attack is typical of brittle wear.

We note, finally, that when the erosion coefficient is known, it is possible to determine the mass flux of erosion products, but the momentum and energy fluxes are still unknown. Consequently, the influence of the erosion products on the flow of a dusty gas is disregarded in the present study. The erosion problem is understood here to mean the solution of the well-known equations of motion of a dusty gas [5] with the boundary condition (1.4) in specific aerodynamic situations.

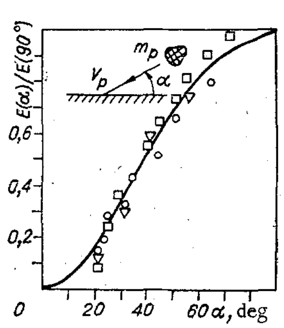

Fig. 1

2. Flow of a Mixture of Gas and Solid Particles Past a Slender Profile

We consider the supersonic flow of a dusty gas past a slender symmetrical profile at zero angle of attack. We assume that the gas is barotropic and that the motion of the particles does not affect the parameters of the gas stream. In this case the equations of motion of the dusty gas are separable into the gasdynamic equations and equations of motion of the particles in the gas flow. By virtue of symmetry we consider only the flow past the upper half of the profile characterized by the equations $y = h_0(x)$, $0 \le x \le b$ and $y(0) = y(b) = 0$. In the absence of erosion the solution of the linearized gasdynamic equations is known [10]:

$$\rho = \rho_\infty\left(1 + \frac{M_\infty^2 v}{\omega u_\infty}\right), \quad u = u_\infty - \frac{v}{\omega}, \quad v = u_\infty h_0'(x - \omega y), \tag{2.1}$$

where $\omega = \sqrt{M_\infty^2 - 1}$; M_∞ is the Mach number; and ρ_∞, u_∞ are the freestream (impingent-flow) values of the density and velocity. The parameters of the stream of particles in the given approximation satisfy the equations [5]

$$\begin{gathered}\frac{\partial}{\partial x}\rho_p u_p + \frac{\partial}{\partial y}\rho_p v_p = 0,\\ \left(u_p\frac{\partial}{\partial x} + v_p\frac{\partial}{\partial y}\right)u_p = \frac{3}{8}\frac{\rho C_D(\mathrm{Re})}{\rho_S r_p}|\mathbf{v} - \mathbf{v}_p|(u - u_p),\\ \left(u_p\frac{\partial}{\partial x} + v_p\frac{\partial}{\partial y}\right)v_p = \frac{3}{8}\frac{\rho C_D(\mathrm{Re})}{\rho_S r_p}|\mathbf{v} - \mathbf{v}_p|(v - v_p),\end{gathered} \tag{2.2}$$

where $C_D(\mathrm{Re})$ is the drag coefficient; the Reynolds number $\mathrm{Re} = 2r_p\rho|\mathbf{v} - \mathbf{v}_p|/\mu$; ρ_S is the density of the particle material; r_p is the particle radius; and μ is the viscosity of the gas. The boundary conditions for Eqs. (2.2) are specified on the Mach line $x = \omega y$ in the form

$$\rho_p = \rho_{p\infty}, \quad u_p = u_\infty, \quad v_p = 0. \tag{2.3}$$

We write the solution of the system (2.2) subject to the condition (2.3) with regard for (2.1) in the form

$$\rho_p = \rho_{p\infty}\left(1 - \frac{M_\infty^2\eta}{\omega}\right)^{-1}, \quad u_p = u_\infty - \frac{v_p}{\omega}, \quad v_p = u_\infty\eta, \tag{2.4}$$

where $\xi = x - \omega y$ and the function $\eta(\xi)$ satisfies the ordinary differential equation

$$\eta' = \frac{3}{8}\frac{\rho M_\infty C_D(\mathrm{Re})|h_0'(\xi) - \eta|(h_0'(\xi) - \eta)}{\rho_S\omega r_p(1 - M_\infty^2\eta/\omega)} \tag{2.5}$$

subject to the initial data $\eta(0) = 0$. The Reynolds number in (2.5) is given by the expression

$$\mathrm{Re} = \frac{2r_p u_\infty M_\infty\rho}{\mu\omega}|h_0'(\xi) - \eta|.$$

Assuming that the particles move through the gas in the Stokes regime and restricting the problem to the linear approximation in Eq. (2.5), we obtain

$$\eta' = l_p^{-1}(h_0'(\xi) - \eta). \tag{2.6}$$

Here $l_p = 2\rho_S\omega r_p^2 u_\infty/M_\infty\mu_\infty$ is the length of the particle relaxation zone.

Inasmuch as the linear velocities of the surface of the profile form a small quantity $\sim\rho_p/\rho_* \lesssim 10^{-3}$ relative to the characteristic amplitude $u_\infty h_0'$ of the perturbations of the gas stream velocity, we can neglect this additional perturbation in solving the gasdynamic problem, i.e., in (2.1) and (2.6) we can set $h = h(\xi, t)$. Using the boundary condition (1.4) and expression (1.7), we have

$$\partial h/\partial t = (dx_0/dt)\partial h/\partial x_1 - G(\partial h/\partial x_1 - \eta)^3, \quad x_1 = x - x_0(t),$$

where $x_0(t)$ is the position of the leading edge and $G = 2\rho_{p\infty}u_\infty^3/3\rho_* H_e$. The resulting equation is solved simultaneously with (2.6) subject to the conditions

$$h(x_1, 0) = h_0(x), \quad h(0, t) = 0.$$

It is readily apparent that the function $x_0(t)$ satisfies the equation

$$\frac{dx_0}{dt} = G\left(\frac{\partial h}{\partial x_1}(0, t)\right)^2, \quad x_0(0) = 0.$$

We have thus reduced the problem of supersonic flow of a mixture of gas and solid particles past a slender profile with erosion to the following: Find the functions h, η, $x_0 \in C^1$ in the domain $D(x_0 \le x \le X;\ 0 \le t \le T)$, satisfying the system of equations

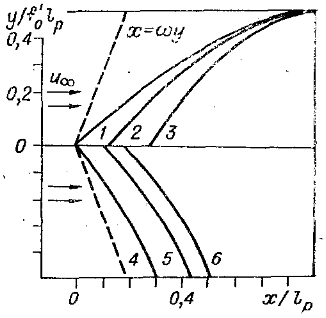

Fig. 2

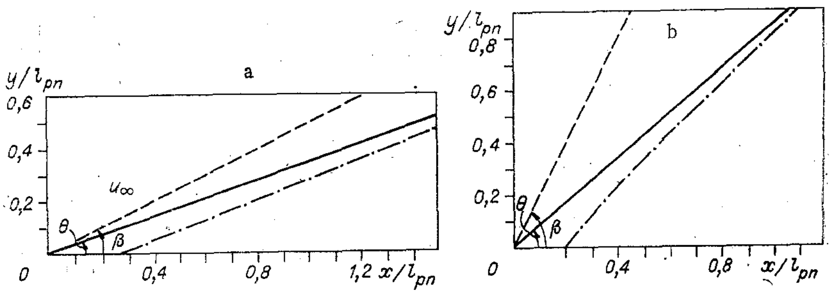

Fig. 3

$$\frac{\partial h}{\partial t}=\frac{dx_0}{dt}\frac{\partial h}{\partial x_1}-G\left(\frac{\partial h}{\partial x_1}-\eta\right)^3,\quad \frac{d\eta}{dx_1}=l_p^{-1}\left(\frac{\partial h}{\partial x_1}-\eta\right),\quad \frac{dx_0}{dt}=G\left(\frac{\partial h}{\partial x_1}(0,\,t)\right)^2$$

and the conditions

$$h(0,\,t)=0,\ \eta(0)=0,\ x_1=x-x_0(t);$$
$$h\,(x_1,\,0)=h_0\,(x),\ x_0\,(0)=0,\ \partial h/\partial x_1\,(x_1,\,0)=h_0'\,(x).$$

The right boundary of the domain $x = X$ does not enter into the statement of the problem and is obtained in the course of the solution as the point at which the difference $(h' - \eta)$ changes sign.

The stated problem has a particular solution of the form

$$h(x_1,\,t)=(1-2At)^{-1/2}f(x_1),\ \eta=(1-2At)^{-1/2}g(x_1),\qquad (2.7)$$
$$x_0\,(t)=-\frac{Gf_0'^2}{2A}\ln{(1-2At)}.$$

Here A is the constant of integration, and the functions $f(x_1)$ and $g(x_1)$ satisfy the equations

$$Gf_0'^2f'-Af=G\,(f'-g)^3,\ f\,(0)=0,\ l_pg'=f'-g,\ g(0)=0,\ f_0'=f'\,(0).\qquad (2.8)$$

The system of equations (2.8) describes the property of profiles with a given slope and curvature:

$$h_0'\,(0)=f_0',\ h_0''\,(0)=3f_0'/2l_p-A/2Gf_0'.$$

It follows from this solution that in erosion a convex profile $[h_0''(0) < 0,\ A > 0]$ becomes even more convex, and its wear rate increases with time. Conversely, any concave profile, for which $h_0''(0) > 3f_0'/2l_p$, flattens during abrasion, and its wear rate diminishes with time. These characteristic attributes of the abrasion (erosion) of a slender profile are shown in Fig. 2. For clarity the longitudinal scale has been changed by the factor l_p, and the transverse scale by l_pf_0'. Curves 1-3 correspond to the locus of the surface of a convex profile in erosion ($At = 0,\ 0.5,\ 1$ respectively), and curves 4-6 to the same for a concave profile with the opposite sign of the parameter A ($|A|t = 0,\ 0.5,\ 1$ respectively). The dashed lines represent the zero Mach line.

3. Erosion of a Wedge in a Supersonic Flow

We now consider the problem of the flow of a gas with entrained solid particles past a wedge. We consider the wedge to be slender and the freestream velocity u_∞ to be such that the shock wave will be attached. We denote the wedge angle by 2θ, as shown in Fig. 3a. We once again invoke the approximation where the motion of the solid particles does not affect the parameters of the gas stream. Following the well-known results of [10], we write the solution for the gas flow behind a shock wave in the form

$$\varepsilon = \rho_\infty/\rho,\ u = u_\infty \cos\beta \cos\theta/\cos(\beta - \theta),\ v = u_\infty \cos\beta \sin\theta/\cos(\beta-\theta). \tag{3.1}$$

The parameters of the motion of the solid particles satisfy the system of equations (2.2), for which the following conditions must be satisfied on the shock line $y = x \tan\beta$:

$$\rho_p = \rho_{p\infty},\ u_p = u_\infty,\ v_p = 0. \tag{3.2}$$

This means that the impingent flow of the gas and particles is in velocity equilibrium.

We write the solution of the system of equations (2.2) subject to the boundary conditions (3.2) in the form

$$\rho_p = \rho_{p\infty}/(1 - (1-\varepsilon)\eta),\ u_p = u_\infty - (u_\infty - u)\eta,\ v_p = v\eta,$$

where the function $\eta(\xi)$ satisfies the equation

$$\eta' = \frac{3}{8}\frac{\rho C_D(\mathrm{Re})\sin\theta \operatorname{ctg}\beta}{\rho_S r_p \cos(\beta-\theta)}\frac{(1-\eta)^2}{1-(1-\varepsilon)\eta},\quad \eta(0) = 0. \tag{3.3}$$

Here $\mathrm{Re} = \mathrm{Re}_0(1-\eta)$; $\mathrm{Re}_0 = 2r_p\rho v/\mu \cos\beta$; $\xi = x \operatorname{tg}\beta - y$. Next, we use a power-law approximation for the drag coefficient: $C_D(\mathrm{Re}) = C_n \mathrm{Re}^{-n}$. In this case the solution of Eq. (3.3) has the form

$$\xi = l_{pn}\left\{\frac{\varepsilon}{1-n}\left[(1-\eta)^{n-1} - 1\right] + \frac{1-\varepsilon}{n}\left[1-(1-\eta)^n\right]\right\},$$

where

$$l_{pn} = \frac{8}{3}\frac{\rho_S \mathrm{Re}_0^n \cos(\beta-\theta)}{\rho C_n \sin\theta \operatorname{ctg}\beta} r_p.$$

It is important to note that for $n = 0.5$ the solution coincides with the well-known solution given in [11]. Figure 4 shows the results of calculations of the function $\eta(\xi)$ for various drag laws and $\varepsilon = 0.32$ (curves 1-3 correspond to $n = 0, 0.5, 1$). Of the two parameters n and l_{pn}, the relaxation length is decisive. Thus, if the drag law is chosen with uncertainty, but the relaxation length is correctly specified, the maximum error of determination of the particle-stream parameters will not be greater than 25%.

We consider the wedge erosion problem in the linear approximation. We choose the erosion coefficient in the form (1.7). Substituting the resulting solutions into expression (1.4) and integrating, we obtain

$$h(y, t) = y \operatorname{ctg}\theta + \frac{\rho_{p\infty}u_\infty}{\rho_*}\left(\frac{u_\infty^2}{2H_e}\frac{4}{3}\sin^2\theta\right)\frac{(1-\eta)^3}{1-(1-\varepsilon)\eta}t, \tag{3.4}$$

where the function $\eta(\xi)$ is calculated on the surface of the wedge, i.e., for $\xi = y(\operatorname{tg}\beta \operatorname{ctg}\theta - 1)$.

Inasmuch as $\eta(\xi)$ varies monotonically over the surface of the wedge, the maximum erosion will take place at the bow of the wedge. The wear rate of the leading edge is

$$\frac{dx_0}{dt} = \frac{\rho_{p\infty}}{\rho_*}\left(\frac{u_\infty^2}{2H_e}\frac{4}{3}\sin^2\theta\right)u_\infty.$$

Figure 3a shows the results of calculations of the erosion of a slender wedge for the parameters $\theta = 19°$, $M_\infty = 5$, $\dot{x}_0 t = 0.3l_p$, $n = 0.5$. The solid line indicates the locus of the surface of the wedge prior to the start of erosion, and the dot-dash line represents its subsequent variation. The magnitude of the wear decreases practically linearly with distance from the leading edge, because the parameter ξ/l_p varies insignificantly here and does not exceed 0.2. Figure 3b shows the erosion of the surface of a wedge with vertex (bow) angle $\theta = 40°$, $M_\infty = 5$, and $n = 0.5$. The nonuniform wear along the surface is distinctly visible. The dashed lines in Figs. 3a and 3b indicate the locus of the shock front prior to the start of erosion.

Using Eq. (1.5) and the solution obtained here, we calculate the average erosion coefficient according to the expression

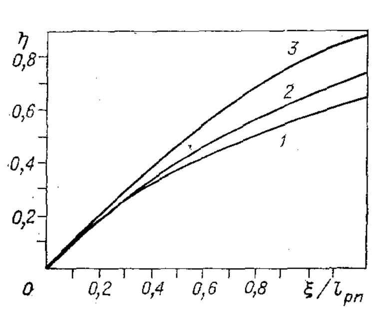

Fig. 4

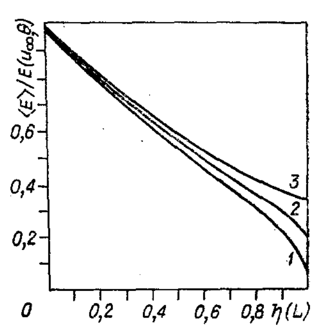

Fig. 5

$$\langle E\rangle = E(u_\infty, \theta)\frac{n}{2+n}\left[\frac{1-(1-\eta)^{2+n}}{1-(1-\eta)^n}\right]_{\xi=L},$$

in which $L = H(\mathrm{tg}\,\beta\ \mathrm{ctg}\,\theta - 1)$, and H is the height of the wedge. It is evident from the resulting relation that edge effects greatly distort the behavior of the erosion coefficient in its averaging, because u_∞ and θ affect the value of $\eta(\xi)$ at the edge point $\xi = L$ (Fig. 5; curves 1-3 correspond to $n = 0, 0.5, 1$). Consequently, the measurements must be performed with on very long wedges ($L \gg l_p$) or on very short ones ($L \ll l_p$).

It follows from the results in Figs. 3a and b that wedges are blunted by erosion. The slope angle of the wedge surface in the vicinity of the leading edge is determined from the relation

$$\mathrm{tg}\,\theta_1 = \mathrm{tg}\,\theta\left[1 + \frac{\dot{x}_0 t}{l_p}(2+\varepsilon)\,(\mathrm{tg}\,\beta - \mathrm{tg}\,\theta)\right].$$

The perturbation of the parameters of the gas stream will be small if $\theta_1 - \theta \ll \theta$. This condition holds if $\tau = \dot{x}_0 t/l_p \ll 1$, so that the error of the linear theory is $O(\tau^2)$. Equation (3.4) therefore has the form

$$h(y, t) = h(y, 0) + [l_p(1-\eta)^3/(1-(1-\varepsilon)\eta)]\tau + l_p O(\tau^2),$$

where the error of the particular approximation is indicated explicitly.

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